

# ABU DHABI POLYTECHNIC ACADEMIC SUPPORT DEPARTMENT

## MATH 1010 – Calculus I

Final Exam Sem.2 - 2024/2025 2 hours

#### Calculators are allowed

No additional materials are required

STUDENT NAME				
STUDENT NUMBER				
		DEPARTMENT ASD		
	Please circle your CRN#:			
	Dr. Bassem: 4493, 4512, 4558, 462	29 Dr. Georgios: 4275, 4380		
	Mr. Tinashe: 4336, 4433, 4591	Dr. Yasser: 4628		

#### READ THESE INSTRUCTIONS CAREFULLY

Write your *name*, *number*, CRN and department **clearly** in the boxes above.

Answer **all** questions.

Show **all** your working and use appropriate **units.** Otherwise, you may lose marks.

You may use a pencil for all your work.

Answers that are not **clearly readable**, if any, will not be marked.

- All mobile devices are not allowed during examination.
- Abu Dhabi Polytechnic considers cheating or attempting to cheat a serious offense that will result in disciplinary action taken against involved individuals.

Part	Score	
Part A.I	/21	
Part A.II	/12	
Part A.III	/6	
Part B.I	/20	
Part B.II	/10	
Part B.III	/11	

Total	/80
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#### Part A.I: CLOs 2,3 (21 grades)

Find the derivative.

1) 
$$\frac{1}{2}x^8 - \frac{1}{3}x^3$$
  
A)  $\frac{1}{2}x^7 - \frac{1}{3}x^2$ 
B)  $4x^9 - x^4$ 
C)  $4x^8 - x^3$ 
D)  $4x^7 - x^2$ 

Find the derivative of the function.

2) 
$$f(x) = (3x^4 + 8)^2$$
  
A)  $6x^4 + 16$  B)  $144x^{15} + 96x^3$  C)  $72x^7 + 192x^3$  D)  $9x^{16} + 64$ 

Find dy/dx by implicit differentiation. If applicable, express the result in terms of x and y.

3) 
$$7y^2 + 3x^2 - 11 = 0$$
  
A)  $\frac{-3x}{7}$ 
B)  $\frac{-3x}{7y}$ 
C)  $\frac{-3x^2}{14y}$ 
D)  $\frac{-6x + 11}{14y}$ 

Find the indicated derivative of the function.

4) 
$$f'''(x)$$
 for  $f(x) = (x + 1)^{-1}$   
A)  $6(x + 1)^{-3}$  B)  $6(x + 1)^{-4}$  C)  $-6(x + 1)^{-3}$  D)  $-6(x + 1)^{-4}$ 

Assume that all variables are implicit functions of time t. Find the indicated rate.

Find those values of x for which the given function is increasing and those values of x for which it is decreasing. 7.2 5.

6) 
$$y = 7x^2 - 5x$$
  
A) Increasing for  $x > \frac{5}{14}$ , decreasing for  $x < \frac{5}{14}$   
B) Increasing for  $x < \frac{5}{14}$ , decreasing for  $x > \frac{5}{14}$   
C) Increasing for  $x > -\frac{5}{14}$ , decreasing for  $x < -\frac{5}{14}$   
D) Increasing for  $x > \frac{5}{7}$ , decreasing for  $x < \frac{5}{7}$ 

Find any relative maximum or minimum points of the given function.

7) 
$$y = 4x^2 - 24x + 32$$
B) Minimum at (-4, 3)A) Minimum at (-4, 3)B) Minimum at (3, -4)C) Maximum at (4, -3)D) Maximum at (-3, 4)

7)

2)

5)

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#### Part A.II: CLOs 4,5 (12 grades)

Integrate the given expression.

8) 
$$\int (6x^2 + 1) dx$$
  
A)  $2x^3 + C$   
B)  $12x + C$   
C)  $x + C$   
D)  $2x^3 + x + C$ 

Find the integral.

9) 
$$\int \sin^3 x \cos x \, dx$$
  
A)  $\frac{\sin^4 x}{3} + C$   
B)  $\frac{\sin^4 x}{4} + C$   
C)  $\frac{\sin^3 x}{3} + C$   
D)  $\frac{\sin^3 x}{4} + C$ 

Integrate the function.

10) 
$$\int x^4 e^{-x^5} dx$$
  
A)  $e^{-x^5} + C$ 
B)  $-\frac{1}{5}e^{-x^5} + C$ 
C)  $-5e^{-x^6} + C$ 
D)  $-\frac{1}{5}e^{-x^6} + C$ 

11)

11) 
$$\int \frac{9e^{9x} dx}{e^{9x} + 1}$$
  
A)  $\ln(e^{9x} + 1) + C$   
B)  $e^{9x} \ln(e^{9x} + 1) + C$   
C)  $9 \ln(e^{9x} + 1) + C$   
D)  $\frac{1}{(e^{9x} + 1)^2} + C$ 

## Part A.III: CLOs 1,6 (6 grades)

Find the exact area under the $12$ ) y = 2x + 7; between	12)			
A) 26	B) 18	C) 52	D) 9	· · · · · · · · · · · · · · · · · · ·
Use L'Hospital's rule to find 13) $\lim_{x \to 0} \frac{\cos 3x - 1}{x^2}$	the limit.			13)
A) 0	B) $\frac{3}{2}$	C) $-\frac{9}{2}$	<b>D)</b> ∞	

#### Part B.I: CLOs 2, 3 (20 grades)

#### **Question 1 (12 grades)**

1) Find the derivative of:  $y = x^3 \sin(x)$ .

2) Find the derivative of:  $y = \ln(e^x - 5)$ .

3) Find the expression of acceleration a(t), if the velocity is:  $v(t) = \sqrt{t^3 - 2t}$ .

4) Find the rate of change of efficiency  $E(T) = 100 \left(\frac{T}{T+300}\right)$ , where T is the temperature.

#### Question 2 (4 grades)

Find the <u>equation</u> of the tangent line to: y = cos(x) + 2x at x = 0.

#### **Question 3 (4 grades)**

A particle moves along a path defined by the parametric equations:

 $x(t) = -t^2 + 2t$  and  $y(t) = t^2 + 3t$  where x(t) and y(t) are in meters, and t is in seconds.

a) Find the components of the velocity vector  $v_x(t)$  and  $v_y(t)$  at any time t.

b) Find the magnitude and direction of the velocity at t = 2 seconds.

#### Part B.II: CLOs 4, 5 (10 grades)

#### Question 1 (6 grades)

Find the integrals below:

1) 
$$\int 5x \cos{(x^2)} dx =$$

2) 
$$\int \sqrt{2x+5} \, dx =$$

#### Question 2 (4 grades)

Find the integral  $\int \frac{1}{(2x+1)^n} dx$  for the value of n below:

a) For 
$$n = 2$$
:

b) For n = 1:

#### Part B.III: CLO 6 (11 grades)

#### Question 1 (5 grades)

Find the area bounded by y = 7x and y = 2x from x = 1 to x = 3.

### Question 2 (6 grades)

a) Find the expression of the displacement s(t) if the velocity  $v(t) = t^2 + 3\pi$ , where the time t is in seconds and s(t) is in meter, given that s = 7 when t = 0.

b) Find the values of the velocity and displacement at t = 3 seconds.

Derivatives	Integrals
$(u^n)' = nu^{n-1}u'$	$\int (du + dv) = u + v + c$
$(\sin u)' = u' \cos u$	$\int u^n du = \frac{u^{n+1}}{n+1} + c  (n \neq -1)$
$(\cos u)' = -u' \sin u$	$\int \frac{1}{u} du = \ln  u  + c$
$(\ln u)' = \frac{u'}{u}$	$\int \sin u  du = -\cos u + c$
$(e^u)' = u'e^u$	$\int \cos u  du = \sin u + c$
$(u \pm v)' = u' \pm v'$	$\int e^u  du = e^u + c$
$(u \cdot v)' = u' \cdot v + u \cdot v'$	Area under the curve $= \int_a^b y  dx$
$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$	Area between curves = $\int_{a}^{b} (y_{\text{High}} - y_{\text{Low}}) dx$

# Calculus - I - Formulae

• The equation of the line passing through  $(x_1, y_1)$  and having a slope m is given by:

$$y - y_1 = m(x - x_1)$$

• Magnitude of the resultant of a vector V is given by:

$$V=\sqrt{V_x^2+V_y^2}$$

• Reference angle of the resultant of vector V is given by:

$$\theta_{\rm ref} = \tan^{-1} \left| \frac{V_y}{V_x} \right|$$

Quadrant-specific Direction  $(\theta)$ :

$$\theta = \begin{cases} \theta_{\rm ref} & \text{if Quadrant I} (V_x > 0, V_y > 0) \\ 180^\circ - \theta_{\rm ref} & \text{if Quadrant II} (V_x < 0, V_y > 0) \\ 180^\circ + \theta_{\rm ref} & \text{if Quadrant III} (V_x < 0, V_y < 0) \\ 360^\circ - \theta_{\rm ref} & \text{if Quadrant IV} (V_x > 0, V_y < 0) \end{cases}$$